Answers

Chapter 5 Review Packet

1. What are the 4 conditions of the Binomial Setting?
   a. n observations (n = fixed number)
   b. prob. of success (p) is same for each observation
   c. all observations are independent
   d. each observation can only be a success or failure

2. What is a sample proportion?

$$\frac{\#\text{ of successes}}{\text{total obs.}} = \frac{X}{n} = \hat{p}$$

3. Circle the correct answers in each sentence
   a. p is a (parameter/statistic) and it describes a (sample/population)
   b. $\hat{p}$ is a (parameter/statistic) and it describes a (sample/population)
   c. $\bar{x}$ is a (parameter/statistic) and it describes a (sample/population)
   d. μ is a (parameter/statistic) and it describes a (sample/population)

4. I have a binomial distribution of a random variable X with B(300, 0.35).
   a. What are n and p? $n = 300$ $p = 0.35$
   b. What is the mean and standard deviation of X?

   $\mu_X = n \cdot p = 105$ $\sigma_X = \sqrt{n \cdot p \cdot (1-p)} \approx 8.2614$

5. What test must a binomial distribution pass in order to approximate it with the normal distribution? $n \cdot p \geq 10$ and $n \cdot (1-p) \geq 10$

6. Finish the sentence:

   Averages are more normal & less variable than individual observations

7. State the Central Limit Theorem in your own words
   Take a sample size n from a population, measure a r.v. X, and take the mean $\bar{x}$ of all n of the X's. As n increases, the sampling distribution of $\bar{x}$ becomes more normal.

8. How does the Central Limit Theorem differ from the Law of Large Numbers?
   Law of Large numbers says that the overall average of n obs becomes closer to the pop. mean (μ) as n increases.
   C.L.T. deals with the sampling distr. of averages of repeated samples of size n.

9. What are the formulas for the mean and standard deviation of each of the following:

On formula sheet!

a. Binomial random variable (X)

$\mu_X = n \cdot p \quad \sigma_X = \sqrt{n \cdot p \cdot (1-p)}$

b. Sample proportion (p)

$\mu_{\hat{p}} = p \quad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

c. Sample mean (x)

$\mu_{\bar{x}} = \mu_{(population)} \quad \sigma_{\bar{x}} = \frac{\sigma_{population}}{\sqrt{n}}$

10. I have a non-normal population, from which I want to take sample means. How large must my sample size be in order to say that the distribution of my sample mean is normal?

$n \geq 30$

For the following problems, show all work. If you use calculator functions to calculate probabilities, show probability notation and calculator work.

11. Experts say that 10% of all American adults agree that schools should be allowed to use physical forms of discipline again. A national opinion poll is taken for an SRS of 60 people. Assume that X (the number of people that agree) is a binomial random variable

p

n

$n \cdot p = 6 \ngeq 10$
$n(1-p) = 50 \geq 10$ ✓

doesn't pass check

binomcdf
binompdf

a. What is the expected number of people that will agree? $\mu_X = n \cdot p = (0.1)(60) = 6$

$\sigma_X = \sqrt{(0.1)(0.9)(60)} \approx 2.3238$

b. What is the probability that exactly 10 people agree?

$P(X=10) = \text{binompdf}(60, 0.1, 10) = 0.03886$

c. What is the probability that greater than 15 people agree?

$P(X>15) = 1 - \text{binomcdf}(60, 0.1, 15) = 0.00020139$

d. What is the probability that 4 people or less agree?

$P(X \leq 4) = \text{binomcdf}(60, 0.1, 4) = 0.27099$

e. What is the probability that 6 or more people agree?

$P(X \geq 6) = 1 - \text{binomcdf}(60, 0.1, 5) = 0.56283$

f. What is the probability that less than 12 people agree?

$P(X < 12) = \text{binomcdf}(60, 0.1, 11) = 0.9854$

g. What is the probability that at least 5 and less than 9 people agree?

$P(5 \leq X < 9) = P(X \leq 8) - P(X \leq 4)$

$= \text{binomcdf}(60, 0.1, 8) - \text{binomcdf}(60, 0.1, 4)$

$= 0.85836 - 0.27099$

$= 0.58737$

12. A university (better known for its basketball program rather than its academics) claims that 80% of its basketball players get degrees (within 5 years). An investigation looks at the fate of 85 basketball players that entered the program over a period of several years.

$p = 0.80$
$n = 85$

$n \cdot p \geq 10$
$n \cdot (1-p) \geq 10$
$(85)(0.8) = 68 \geq 10$ ✓
$(85)(0.2) = 17 \geq 10$ ✓

a. What is the expected (mean) percent of players that get degrees? $\mu_{\hat{p}} = p = 0.80$

b. What is the probability that less than 75% of players get degrees? $\sigma_{\hat{p}} = \sqrt{\frac{0.8 \cdot 0.2}{85}} = 0.043386$

$P(\hat{p} < 0.75) = \text{normalcdf}\left(-E99, 0.75, 0.8, \sqrt{\frac{(0.8)(0.2)}{85}}\right) = 0.1245695$

c. What is the probability that more than 82% of players get degrees?

$P(\hat{p} > 0.82) = \text{normalcdf}\left(0.82, E99, 0.8, \sqrt{\frac{(0.8)(0.2)}{85}}\right) = 0.3224$

d. What is the probability that the percent of players that get degrees is within 5% of the mean?

$P(0.75 \leq \hat{p} \leq 0.85) = \text{normalcdf}\left(0.75, 0.85, 0.8, \sqrt{\frac{(0.8)(0.2)}{85}}\right) = 0.7509$

13. Billy takes a multiple-choice test. There are 115 questions. Billy knows some of the material (but not much!), so his probability of getting a right answer is 0.57.

$n = 115$
$p = 0.57$

$n \cdot p = 65.55 \geq 10$ ✓
$n(1-p) = 49.45 \geq 10$ ✓
norm. approx.

a. What is the expected number of questions that Billy ~~will get~~ right?

$\mu_X = n \cdot p = (115)(0.57) = 65.55$

$\sigma_X = \sqrt{0.57 \cdot 0.43 \cdot 115} = 5.309$

b. What is the expected score (percent) that Billy will get on the test?

$\mu_{\hat{p}} = p = 0.57 = 57\%$

$\sigma_{\hat{p}} = \sqrt{\frac{(0.57)(0.43)}{115}} = 0.046166$

c. What is the probability that Billy will get more than 85 questions right?

$P(X > 85) = \text{normalcdf}(85, E99, 65.55, 5.309) = 0.00012439$

d. What is the probability that Billy will get between 65 and 80 questions right?

$P(65 \leq X \leq 80) = \text{normalcdf}(65, 80, 65.55, 5.309) = 0.538$

e. What is the probability that Billy will get less than 63 ~~questions right~~?

$P(X < 63) = \text{normalcdf}(-E99, 63, 65.55, 5.309) = 0.3155$

f. What is the probability that Billy will fail (failing is below a 60%)?

$P(\hat{p} < 0.60) = \text{normalcdf}\left(-E99, 0.60, 0.57, \sqrt{\frac{(0.57)(0.43)}{115}}\right) = 0.742098$

g. What is the probability that Billy will get between a 75% and an 85%?

$P(0.75 \leq \hat{p} \leq 0.85) = \text{normalcdf}\left(0.75, 0.85, 0.57, \sqrt{\frac{(0.57)(0.43)}{115}}\right) = 0.00004832$

h. What is the probability that Billy will get an A- or above (an A- is a 90%)

$P(\hat{p} \geq 0.90) = \text{normalcdf}\left(0.90, E99, 0.57, \sqrt{\frac{(0.57)(0.43)}{115}}\right) = 4.43 \times 10^{-13}$

almost 0!

$\mu = 2.2$
$\sigma = 1.4$

14. The number of accidents per week at a hazardous intersection varies with mean 2.2 and standard deviation 1.4. The distribution takes only whole number values, so it is certainly not normal. Let $\bar{x}$ be the number of mean accidents per week at the intersection during a year (52 weeks).

Is $n \geq 30$? $52 \geq 30$ ✓

a. What is the approximate distribution of $\bar{x}$ (according to the Central Limit Theorem)? Since $n \geq 30$,

$$\bar{X} \sim N(2.2, 0.19415)$$

b. What is the probability that $\bar{x}$ is less than 2?

$$P(\bar{X} < 2) = \text{normalcdf}(-E99, 2, 2.2, 0.19415) = 0.15147$$

c. What is the probability that there are fewer than 100 accidents at the intersection in a year? *(Hint: restate this event in terms of $\bar{x}$)*

$$\frac{100 \text{ accidents/yr}}{52 \text{ weeks/yr}} = 1.9231 \text{ accidents/wk.}$$

$$P(X < 1.9231) = \text{normalcdf}(-E99, 1.9231, 2.2, 0.19415) = 0.0769$$

15. The scores on the SATs at a large high school are normally distributed with a mean of 950 and a standard deviation of 85.

$\mu_{\bar{X}} = 950$
$\sigma_{\bar{X}} = \frac{85}{\sqrt{100}} = 8.5$

a. What is the probability that a randomly selected student from the high school will have a score of 1000 or above?

$$P(X > 1000) = \text{normalcdf}(1000, E99, 950, 85) = 0.2782$$

b. What is the probability that a randomly selected student from the high school will have a score of 1250?

continuous! $\text{normalcdf}(1250, 1250, 950, 85) = 0$

$$P(X = 1250) = 0$$

c. What is the probability that a randomly selected student from the high school will have a score of 800 or below?

$$P(X \leq 800) = \text{normalcdf}(-E99, 800, 950, 85) = 0.03881$$

d. If we take a sample of 100 students from the high school, what is the distribution of $\bar{x}$, the mean of their scores on the SATs? NORMAL

$$\mu_{\bar{X}} = 950 \quad \sigma_{\bar{X}} = 8.5$$

e. What is the probability that a randomly selected sample of 100 students from the high school will have a mean score of 1200 or above?

$$P(\bar{X} \geq 1200) = \text{normalcdf}(1200, E99, 950, 8.5) = 0$$

f. What is the probability that a randomly selected sample of 100 students from the high school will have a mean score of below 1050?

$$P(\bar{X} < 1050) = \text{normalcdf}(-E99, 1050, 950, 8.5) = 1$$

g. What mean score will have 75% of the students below it?

$$P(\bar{X} < K) = 0.75 \quad \text{invNorm}(0.75, 950, 8.5) = 955.733$$

16. We want to look at the number of hours of TV watched per day by each gender. We are concentrating our efforts on working people ages 18-25. The distribution of the number of hours watched per day is normally distributed for both genders. The distribution for the men is N(2.25, 0.4), and for the women N(1.75, 0.5). The hours watched by the men is independent of the hours watched by the women.

$M: N(2.25, 0.4)$
$W: N(1.75, 0.5)$

a. What is the distribution for the difference between the men and the women (men – women)?

$\mu_{M-W} = \mu_M - \mu_W = 2.25 - 1.75 = 0.5$

$\sigma^2_{M-W} = \sigma^2_M + \sigma^2_W = (0.4)^2 + (0.5)^2 = 0.41$

$\sigma_{M-W} = \sqrt{\sigma^2_{M-W}} = \sqrt{0.41} = 0.6403$

$N(0.5, 0.6403)$

b. What is the probability that the difference between the men and the women is more than 0.75 hours?

$P(M-W > 0.75) = \text{normalcdf}(0.75, E99, 0.5, 0.6403) = 0.3481$

$\bar{M} = N(2.25, 0.4/\sqrt{20})$
$\bar{W} = N(1.75, 0.5/\sqrt{20})$

c. If I take a random sample of 20 men and 20 women, what is the sampling distribution of the average difference between the men and the women?

$\mu_{\bar{M}-\bar{W}} = 0.5 \quad \sigma^2_{\bar{M}-\bar{W}} = \sigma^2_{\bar{M}} + \sigma^2_{\bar{W}} = \left(\frac{0.4}{\sqrt{20}}\right)^2 + \left(\frac{0.5}{\sqrt{20}}\right)^2 = 0.0205 \quad \sigma = \sqrt{0.0205} = 0.14318$

$N(0.5, 0.14318)$

d. What is the probability that the sample mentioned in part (c) has an average difference of 0.75 or more?

$P(\bar{M}-\bar{W} > 0.75) = \text{normalcdf}(0.75, E99, 0.5, 0.14318) = 0.0404$